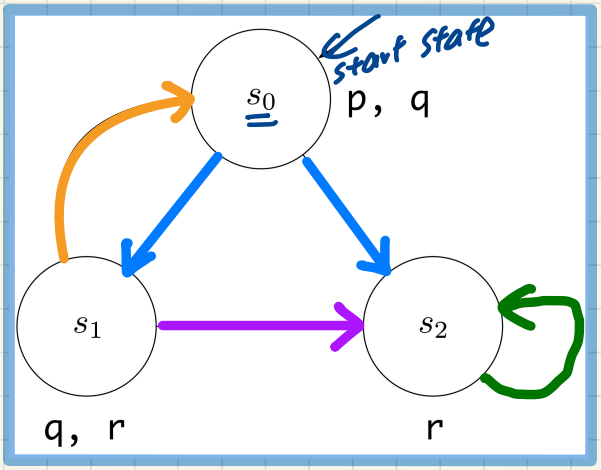
Lecture 14 - March 3

Model Checking

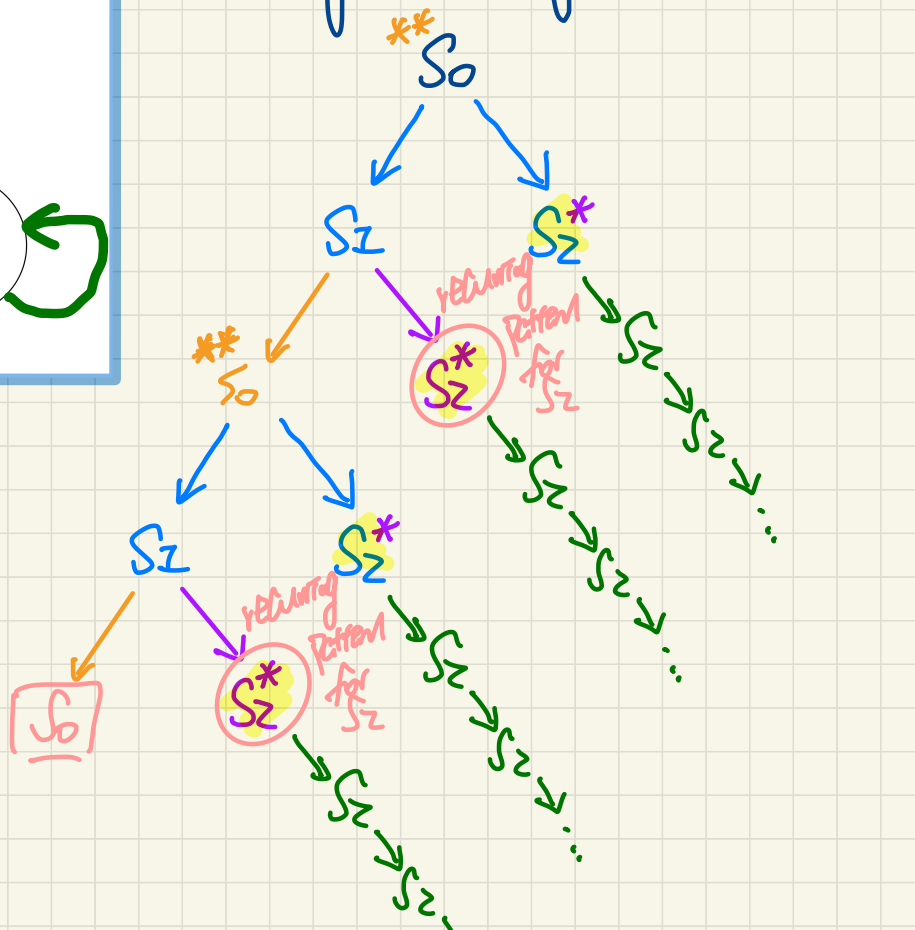
Unfolding/Unwinding Paths
Satisfaction Relations: Path vs. Model
Formulations: X, F, G

Announcements/Reminders

- **ProgTest1** results to be released (by end of Friday)
- **WrittenTest1** guide & examples released
  + **Review Q&A** (Zoom): 7:30pm on Monday, Mar 3
- **Lab3** to be released after WrittenTest1
- Office Hours: 3pm to 4pm, Mon/Tue/Wed/Thu
- TA contact information (on-demand for labs) on eClass

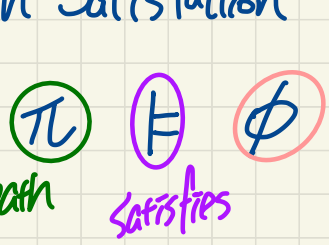


Unfolding/Unwinding (transitions)

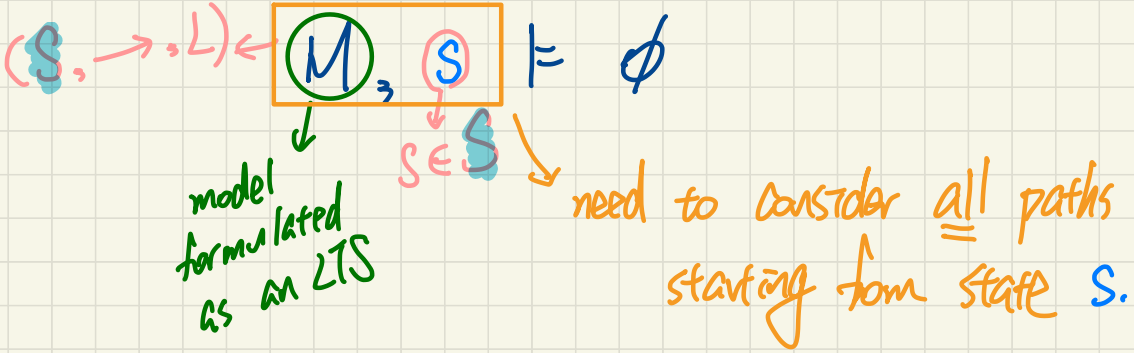


Satisfaction Relations

(1) Path Satisfaction

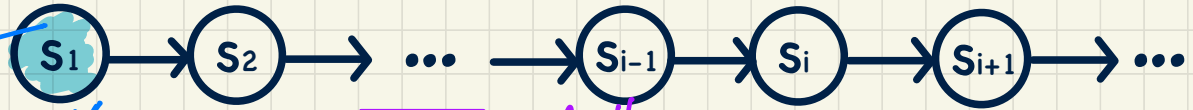


(2) Model Satisfaction



Path Satisfaction: Logical Operations

A path satisfies a proposition
if its initial state ("current state") satisfies it.



first state of π

base cases

$\pi \models p \Leftrightarrow p \in \mathcal{L}(s_1)$ — set of all propositional atoms satisfied in the 1st state (s_1) of π; $p \in \mathcal{L}(s_i)$

$\pi \models \top$ (whether the 1st state of π satisfies $\top$)

$\pi \models \bot$

recursive cases

$\pi \models \neg\phi \Leftrightarrow \neg(\pi \models \phi)$

$\pi \models \phi_1 \wedge \phi_2 \Leftrightarrow \pi \models \phi_1 \wedge \pi \models \phi_2$

$\pi \models \phi_1 \vee \phi_2 \Leftrightarrow \pi \models \phi_1 \vee \pi \models \phi_2$

$\pi \models \phi_1 \Rightarrow \phi_2 \Leftarrow \pi \models \phi_1 \Rightarrow \pi \models \phi_2$

* $\forall i \cdot i \,\%\, 2 = 0 \wedge i > 0 \Rightarrow \pi^i \models p$ (2, 4, 6, ...)

π^i: path starting from the ith component of π

$\forall i \cdot i \in \mathbb{N} \wedge i > 0 \Rightarrow \pi^{2i} \models p$

* Q: Express that **all** the even-numbered states satisfies a proposition p.

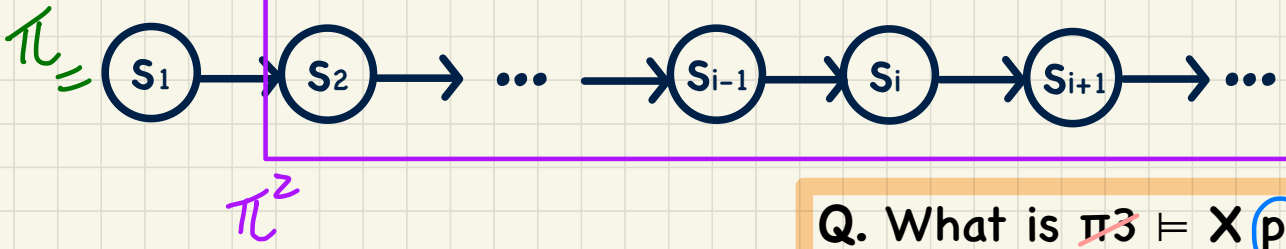
($s_2, s_4, s_6, \ldots$)

Path Satisfaction: Temporal Operations (1)

A path satisfies $X\phi$ (next op.) → nested, arbitrarily complicated LTL formula.

if the next state (of the "current state") satisfies it.

$\pi \models X\phi$ $\quad$ $\pi^2 \models \phi$



$\pi = s_1 \to s_2 \to s_3 \to s_4 \to s_5 \to \cdots$ $\quad$ π^3 $\quad$ $(\pi^3)^2$ $\quad$ π^4

Q. What is $\pi^3 \models X\,p$ checking? (p: prop. atom)

Formulation (over a path)

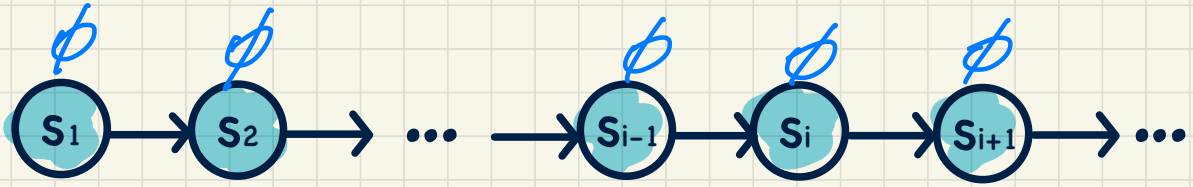
$$\pi \models X\phi \iff \pi^2 \models \phi$$

π^2: a path starting from the 2nd component of ϕ

$$\pi^3 \models Xp \iff (\pi^3)^2 \models p \iff \pi^4 \models p \iff p \in L(s_4)$$

Path Satisfaction: Temporal Operations (2)

A path satisfies $\mathbf{G}\phi$ (Global) → Henceforth, ϕ is true.
if the every state satisfies it.



Formulation (over a path)

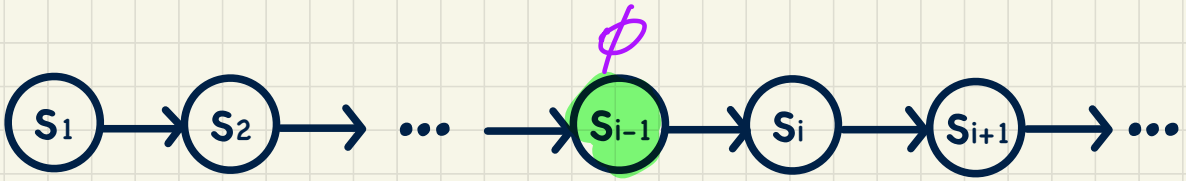
$$\pi \vDash \mathbf{G}\phi \iff \forall i \cdot i \geq 1 \Rightarrow \pi^i \vDash \phi$$

□

$i \in \mathbb{N}$ X
$\because i = 0 \Rightarrow \pi^0$ undefined.

Path Satisfaction: Temporal Operations (3)

A path satisfies $\mathbf{F}\phi$ (Future) ⟶ Eventually, ϕ is true.
if some future state satisfies it.

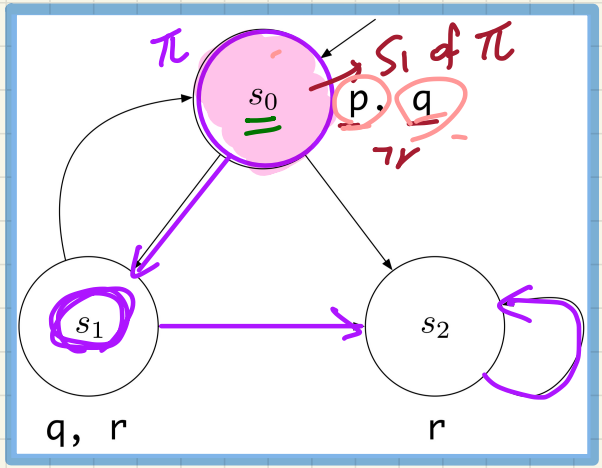


Formulation (over a path)

$$\pi \vDash \boxed{\mathbf{F}}\phi \iff \exists i \cdot i \geq 1 \wedge \pi^i \vDash \phi$$

$\boxed{\mathbf{F}} \equiv \Diamond$

Model vs. Path Satisfaction: Exercises (1.1)



Recall: $\pi \models p \Leftrightarrow p \in L(s_1)$

Say: $\pi = s_0 \to s_1 \to s_2 \to s_2 \to \dots$

- $\pi \models \top$ (T)
- $\pi \not\models \bot$ (T) $\quad \pi \not\models \bot \equiv \neg(\pi \models \bot)$
- $\pi \models p \wedge q \equiv \pi \models p \wedge \pi \models q \equiv$ (T) $\quad p \in L(s_0)$, $q \in L(s_0)$
- $\pi \models p \vee q$ (T)
- $\pi \models p \Rightarrow q \equiv \pi \models p \Rightarrow \pi \models q \equiv$ (T)
- $\pi \models r \equiv r \in L(s_0)$ (F)
- $\pi \models r \Rightarrow p \wedge q \wedge r \equiv$

$\pi^2 \models \top$

$\pi^2 \models \bot$

$\pi^2 \models r \Rightarrow p \wedge q \wedge r$

Exercise: What if we change the LHS to π^2?

$\pi \models r \Rightarrow \pi \models p \wedge q \wedge r$ (F) (T)